

Attempt to solve the following four questions

Student name: _____

Dept. _____ Section _____

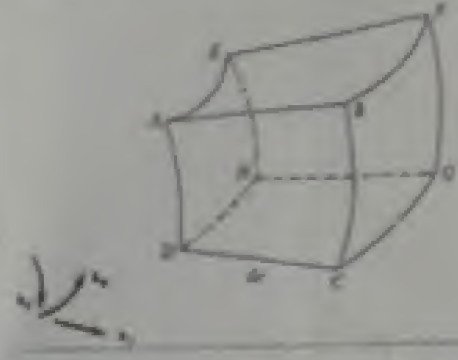
Question 1

Fill the tables with your answers:-

15 Points

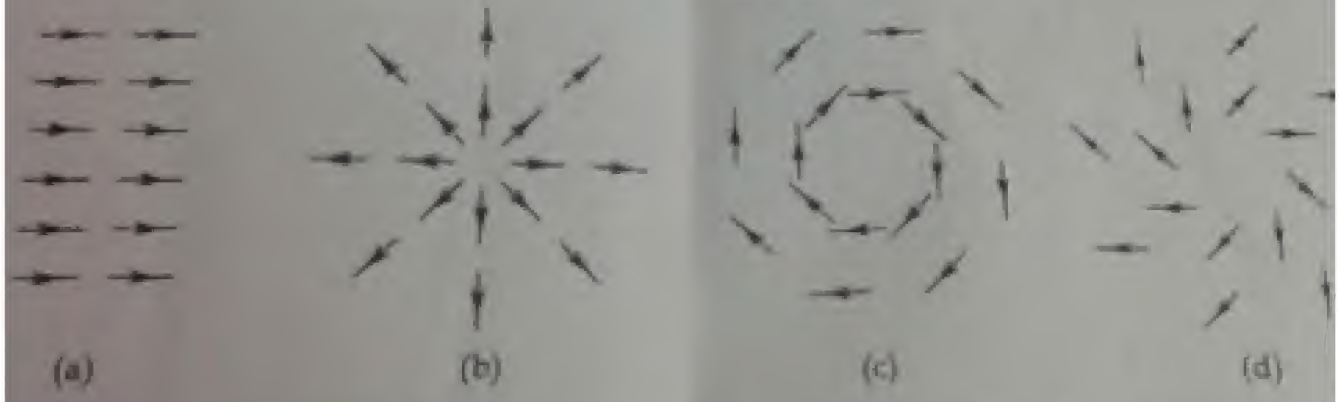
1) A differential volume in spherical coordinates is shown in Figure. For the volume element, select the items in the left column with those in the right column.

- | | |
|------------------------|--|
| (a) dL from A to D | (1) $-r^2 \sin \theta \, d\theta \, d\phi \, dr$ |
| (b) dL from E to A | (2) $-r \sin \theta \, dr \, d\phi \, d\theta$ |
| (c) dL from A to B | (3) $r \, dr \, d\theta \, d\phi$ |
| (d) dS for face EFGH | (4) $dr \, d\theta$ |
| (e) dS for face AEHD | (5) $r \, d\theta \, d\phi$ |
| (f) dS for face ABFE | (6) $r \sin \theta \, d\phi \, d\theta$ |



| Fill this table with your selections | (a) | (b) | (c) | (d) | (e) | (f) |
|--------------------------------------|-----|-----|-----|-----|-----|-----|
| | | | | | | |

2) The following Figure illustrate four categories of vector field \vec{A} . Which of these are vanishing divergence ($\nabla \cdot \vec{A} = 0$) and which are nonvanishing divergence ($\nabla \cdot \vec{A} \neq 0$)?



| Fill this table with your answers | Vector fields | Vanish or nonvanish divergence |
|-----------------------------------|---------------|--------------------------------|
| | (a) | |
| | (b) | |
| | (c) | |
| | (d) | |

Electromagnetic fields
midterm

Question 1

- ① a - 5
b - 6
c - 4
d - 3
e - 1
f - 2

- ② (a) $\nabla \cdot \vec{A} = 0$ (vanishing)
(b) $\nabla \cdot \vec{A} \neq 0$ (non vanishing)
(c) $\nabla \cdot \vec{A} = 0$ (vanishing)
(d) $\nabla \cdot \vec{A} \neq 0$ (non vanishing)

Question 2

(a) $\vec{E} = \vec{E}_Q + \vec{E}_L + \vec{E}_s$

$$\vec{E} = \frac{-500 \times 10^{-9}}{4 \times 10^{-6} \pi^2} \vec{R} + \frac{-10 \times 10^{-9}}{2 \times 10^{-6} \pi^2} \vec{a}_\rho + \frac{-10 \times 10^{-9}}{2 \times 10^{-6} \pi^2} \vec{a}_z$$

$$\vec{R} = -3\vec{a}_x - 4\vec{a}_y \quad R = \sqrt{9+16} = 5$$

$$\vec{\rho} = -3\vec{a}_z \quad \rho = 3$$

$$\vec{a}_\rho = -\vec{a}_z$$

$$\vec{a}_z = \vec{a}_z$$

$$\vec{E} = 3\vec{a}_x + 4\vec{a}_y + \vec{a}_z + \vec{a}_z$$

$$\boxed{\vec{E} = 3\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z \text{ V/m (N/C)}}$$

Question 2

15 Points (10+5)

Consider the following charge distributions in free-space:

- 1) A point charge $Q = -500\pi\epsilon_0$ C located at $(3,4,4)$
- 2) An infinite line $x = 0, z = 9$, carrying line charge density $\rho_L = -10\pi\epsilon_0$ C/m
- 3) An infinite surface $z = -3$ carrying surface charge density $\rho_S = 2\epsilon_0$ C/m²

Calculate: a) the electric field intensity at the point $(0,0,4)$.
b) the total electric flux leaving a sphere of radius 5 m centered at the origin.

$$r > 2 \text{ m}:$$

$$\begin{aligned} Q_{\text{enc}} &= Q_{\text{point}} + Q_{\text{surface}} \\ &= 10 + (-1) \cdot 4\pi (2)^2 \text{ Mc} \\ &= 10 - 16\pi \text{ Mc} \end{aligned}$$

$$Q_{\text{enc}} = \oint \vec{D} \cdot d\vec{s}$$

$$10 - 16\pi = D_r \cdot 4\pi r^2$$

$$\boxed{\vec{D} = \frac{10 - 16\pi}{4\pi r^2} \vec{a}_r \text{ Mc/m}^2}$$

b) at $r = 7$

$$D = 0 \rightarrow Q_{\text{enc}} = 0$$

$$Q_{\text{enc}} = 10 - 16\pi + Q_{\text{volume}} = 0$$

$$Q_{\text{vol}} = 16\pi - 10 \text{ Mc} = \int P_v dv$$

$$Q_{\text{vol}} = \int_0^{2\pi} \int_0^\pi \int_4^6 P_v r^2 \sin\theta dr d\theta d\phi$$

$$= P_v \cdot \frac{r^3}{3} \Big|_4^6 \cdot (-\cos\theta) \Big|_0^\pi \cdot \phi \Big|_0^{2\pi}$$

$$= P_v \cdot \frac{152}{3} \cdot 2 \cdot 2\pi$$

$$= 636.696 P_v$$

$$16\pi - 10 = 636.696 P_v$$

$$\boxed{P_v = 0.063 \text{ Mc/m}^3}$$

Question 3

15 Points (2+3+5+2)

Given that $\vec{E} = \vec{a}_x + z^2\vec{a}_y + 3y^2z\vec{a}_z$ V/m in free-space.

Find: (a) the electric flux density.

(b) the volume charge density at the origin.

(c) the electric flux through a cube of side 2 m and centered at the origin by two methods.

(d) the work done in carrying a 5 mC charge from A(1,2,-4) to B(3,2,-4).

Question 4

Consider the

- a point
- a unif

(a) Calcula

(b) What

in the

111 0

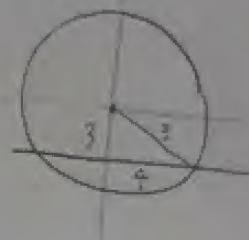
101. ✓

(b) $\Psi = Q_{enc}$

→ the point charge and the line are outside the sphere

$$\rightarrow \Psi = Q_{enc} = D_s A$$
$$= 2 \epsilon_0 \cdot \pi (4^2)$$

$$\boxed{\Psi = 32 \pi \epsilon_0 C} \quad \text{wb}$$



Question 3

(a) $\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 (\hat{a}_x + z^2 \hat{a}_y + 3y^2 z \hat{a}_z) \quad C/m^2$

(b) $\rho_v = \nabla \cdot \vec{D} = 3y^2 \epsilon_0 \quad C/m^3$

at the origin:

$$\boxed{\rho_v = 0}$$

(c) First method:

$$\Psi = Q_{enc} = \int \rho_v dV = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 3y^2 \epsilon_0 dx dy dz$$
$$= \epsilon_0 x \Big|_{-1}^1 y^3 \Big|_{-1}^1 z \Big|_{-1}^1$$
$$= 2 * 2 * 2 * \epsilon_0 = \boxed{8 \epsilon_0 C}$$

Second method:

$$\Psi = Q_{enc} = \int \vec{D} \cdot \vec{ds}$$
$$= \epsilon_0 \left[\int_{x=1} \int_{y=-1}^1 \int_{z=-1}^1 dy dz - \int_{x=-1} \int_{y=-1}^1 \int_{z=-1}^1 dy dz \right.$$
$$+ \int_{y=1} \int_{x=-1}^1 \int_{z=-1}^1 z^2 dx dz - \int_{y=-1} \int_{x=-1}^1 \int_{z=-1}^1 z^2 dx dz$$
$$\left. + \int_{z=1} \int_{x=-1}^1 \int_{y=-1}^1 3y^2 z dx dy - \int_{z=-1} \int_{x=-1}^1 \int_{y=-1}^1 3y^2 z dx dy \right]$$

$\boxed{z=1} \quad \quad \quad \boxed{z=-1}$

Question 4

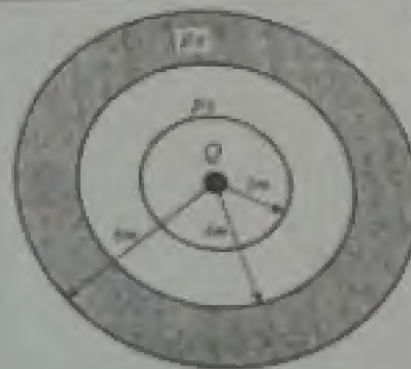
Consider the following charge distribution:

- a point charge of $10\mu\text{C}$ is located at $r = 0$, and
- a uniform surface charge density of $-1\mu\text{C}/\text{m}^2$ at $r = 2$

(a) Calculate the electric flux density D everywhere?

(b) What uniform volume charge density should be established in the region $4 < r < 6$ for D to vanish at $r = 7$?

15 Points (10+5)



$$\begin{aligned}
 Q_{enc} &= 2\epsilon_0 \int_{-1}^1 \int_{-1}^1 3y^2 dx dy \\
 &= 2\epsilon_0 * y^3 \Big|_{-1}^1 * x \Big|_{-1}^1 \\
 &= 2\epsilon_0 * 2 * 2
 \end{aligned}$$

$$\boxed{\psi = 8\epsilon_0 \text{ C}}$$

$$\begin{aligned}
 (d) \quad w &= -q \int \vec{E} \cdot d\vec{L} \\
 &= -9 \times 10^{-3} \int_1^3 1 * dx \\
 &= -5 \times 10^{-3} * 2
 \end{aligned}$$

$$\boxed{w = -10 \text{ mJ}}$$

Question 4

$$\oint \vec{D} \cdot d\vec{s} = \iiint D_r \cdot r^2 \sin\theta \, d\theta \, d\phi = D_r \cdot 4\pi r^2$$

$$\underline{0 < r < 2 \text{ m}}$$

$$Q_{enc} = 10 \text{ nC}$$

$$Q_{enc} = \oint \vec{D} \cdot d\vec{s}$$

$$10 \text{ nC} = D_r \cdot 4\pi r^2$$

$$D_r = \frac{10 \text{ nC}}{4\pi r^2}$$

$$\boxed{\vec{D} = \frac{10}{4\pi r^2} \vec{a}_r \text{ nC/m}^2}$$



Yamou University
Faculty of Engineering
Electrical Power and Machines Engineering Dept.



Mid Term Exam - First Semester No. 1, 2019

Course: (ENR) (ENR) - Electromagnetics (ENR)

Time allowed: 40 minutes

Year: 1st (ENR) (ENR) (ENR)

Date: 10/1/2019

Answer to solve the following five questions

Total Points: 40

Student name:

Date:

Section:

(15 Points (5+10))

Question 1

Consider a distribution of three charges in free space consisting of:

- A point charge $Q_1 = 200 \mu\text{C}$ at $(6, 8, 4)$.
- A uniform line charge density $\rho_L = 200 \mu\text{C/m}$ at $x = 8, z = -4$ and
- A uniform surface charge density $\rho_s = 40 \mu\text{C/m}^2$ at $x = 8$.

Find: (a) The total electric flux leaving a cube of side 10 is centered at the origin.
(b) The total electric flux density at point $(-6, 8, 4)$.

$$\begin{aligned} \psi &= \sum Q \\ &= Q_1 + Q_2 \\ &= 200 \mu\text{C} + 80 \mu\text{C} = 280 \mu\text{C} \end{aligned}$$

$$\begin{aligned} \vec{E} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\ &= \frac{Q_1}{4\pi\epsilon_0 r^2} \hat{r} + \frac{\rho_L}{2\epsilon_0} \hat{r} + \frac{\rho_s}{\epsilon_0} \hat{r} \end{aligned}$$

$$\vec{r} = (8, 0, 4) - (6, 8, 4) = (2, -8, 0)$$

$$r = 10$$

$$\vec{r} = (2, -8, 0) = 10 \hat{r} = +8 \hat{r}$$

$$r = 10$$

$$\hat{r} = \hat{r}$$

$$\hat{r} = \hat{r}$$

$$\begin{aligned} \vec{E} &= \frac{200 \times 10^{-6}}{4\pi \times 10^{-12} \times 10^2} (-6, -8, 0) + \frac{200 \times 10^{-6}}{2 \times 10^{-12} \times 5} \hat{r} \\ &+ \frac{40 \times 10^{-6}}{10^{-12}} \hat{r} \end{aligned}$$

$$\begin{aligned} \vec{E} &= \frac{1}{10} (-6, -8, 0) \\ &+ 2 \hat{r} \\ &+ 4 \hat{r} \end{aligned}$$

$$\vec{E} = \frac{1}{10} (-6, -8, 0) \text{ V/m}$$

| | Answers | Unit |
|-----|---------|------|
| (a) | | |
| (b) | | |

Question 2

15 Points (10+5)

The semicircular ring of charge shown in figure has a radius of 4 m. It carries a uniform line charge of $5\pi\epsilon_0$ C/m. Find:

- the potential at the points A(0,0,0) and B(0,0,3)?
- the work done needed to move a unit positive charge from point A to point B?

$$V_P = \int \frac{\rho_L dL}{4\pi\epsilon_0 R}$$

$$\vec{R} = (0,0,h) - (x,y,0) \\ = -x\hat{a}_x - y\hat{a}_y + h\hat{a}_z$$

$$R = \sqrt{x^2 + y^2 + h^2}$$

$$R = \sqrt{r^2 + h^2}$$

$$R = \sqrt{16 + h^2}$$

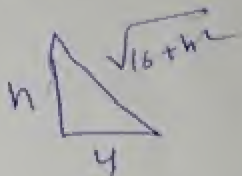
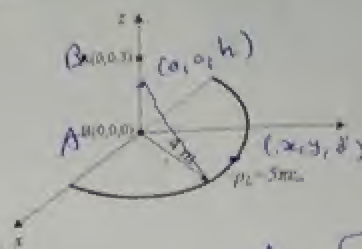
$$V_P = \int_0^\pi \frac{5\pi\epsilon_0 \cdot 4 d\phi}{4\pi\epsilon_0 \sqrt{16 + h^2}}$$

$$= 5 \frac{(2\pi)}{\sqrt{16 + h^2}} = \frac{5\pi}{\sqrt{16 + h^2}}$$

$$V_B = \frac{5}{\sqrt{16 + 3^2}} = \pi V$$

$$V_A = \frac{5\pi}{\sqrt{16 + 0}} = 1.25\pi V$$

$$W_{AB} = Q(V_B - V_A) \\ = \frac{1}{4}\pi J$$



$$\frac{5\pi \times 4}{\sqrt{16 + 3^2}} = \frac{20\pi}{5} = 4\pi$$

| | Answers | Units |
|-----|---------|-------|
| (a) | | |
| (b) | | |

Question 3

15 Points

Given that $\vec{E}_1 = 3\vec{a}_x - 4\vec{a}_y + 5\vec{a}_z$ V/m is the structure shown by the following figure. Find:

- a)
- D_1
- b)
- \vec{E}_2
- c)
- P_2
- d)
- θ_2

$$\vec{E}_{1n} = 5\vec{a}_z \quad \vec{D}_{1n} = \epsilon_0 \epsilon_{r1} 5\vec{a}_z$$

$$\vec{E}_{1t} = 3\vec{a}_x - 4\vec{a}_y$$



$$\vec{E}_{2t} = \vec{E}_{1t} = 3\vec{a}_x - 4\vec{a}_y$$

$$\vec{D}_{2n} = \vec{D}_{1n} = 10\epsilon_0\vec{a}_z$$

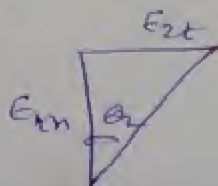
$$\vec{D}_{2t} = \epsilon_0 \epsilon_{r2} \vec{E}_{2t} = (3\vec{a}_x - 4\vec{a}_y)\epsilon_0$$

$$\textcircled{a} \quad \vec{D}_1 = \epsilon_0 \epsilon_{r1} [(3, -4, 5)] \quad \text{C/m}^2 \quad \textcircled{2}$$

$$\textcircled{b} \quad \vec{E}_2 = \frac{\vec{D}_2}{\epsilon_0 \epsilon_{r2}} = (3, -4, 10) \quad \text{V/m} \quad \textcircled{3}$$

$$\textcircled{c} \quad P_2 = \chi_{e2} \epsilon_0 \vec{E}_2 = (\epsilon_{r2} - 1) \epsilon_0 \vec{E}_2 = 0 \quad \textcircled{2}$$

$$\textcircled{d} \quad \theta_2 = \tan^{-1} \frac{E_{2t}}{E_{2n}} \quad \textcircled{3}$$



| | Answers | Units |
|-----|---------|-------|
| (a) | | |
| (b) | | |
| (c) | | |
| (d) | | |

Question 4

15 Points (2+2+2+4+2+3)

An air filled parallel plate capacitor stores $5\mu\text{J}$ electric energy when the voltage applied between its terminals is 100V .

- Find the capacitance?
- If the distance between the plates is 0.8854 mm , find the area of each plate?
- What is the surface charge density on each plate?
- What is the energy density in the capacitor using two methods? 2+2
- If the capacitor is filled by a dielectric material with a relative permittivity of 4, calculate the new capacitance of the capacitor?
- Calculate the polarization of the dielectric used in part e)?

$$W = 5\mu\text{J} = \frac{1}{2} C_0 V_0^2$$

$$V = 100\text{ V}$$

$$\checkmark \text{ a) } C = \frac{2W}{V^2} = 1\text{ nF} \quad \frac{1}{2} \frac{\epsilon_0 A}{d}$$

$$\checkmark \text{ b) } d = ? \quad C = \epsilon_0 \epsilon_r \frac{A}{d} \quad d = \epsilon_0 \frac{A}{C} \quad A = \frac{Cd}{\epsilon_0}$$

$$\checkmark \text{ c) } \rho_s = \frac{Q}{A} = \frac{CV}{A} = 1\text{ M C/m}^2 \quad C = \frac{Q}{V} = \frac{\rho_s A}{V}$$

$$\rho_s = \epsilon_0 E$$

$$\checkmark \text{ d) } W_E = \frac{1}{2} \epsilon_0 E^2 = \frac{W}{Ad} = 0.05644$$

$$\checkmark \text{ e) } C = \frac{\epsilon_0 \epsilon_r A}{d} = 4\text{ nF}$$

$$\checkmark \text{ f) } P = \chi_e \epsilon_0 E$$

$$= (\epsilon_r - 1) \epsilon_0 E$$

$$= 2.998 \times 10^{-6}$$

| | Answers | Units |
|-----|---------|-------|
| (a) | | |
| (b) | | |
| (c) | | |
| (d) | | |
| (e) | | |

Wish you all the best

Prof. Ahmed Shobair and Dr. Sherif Dabour

$$W = \frac{1}{2} QV$$

$$\frac{2W}{V^2} = \frac{QV}{V^2}$$

Student name: _____

Lept. _____

Section _____

10

Consider the following potential field in free-space

$$V = -2x^2 - 3y + 9z \quad \text{mV}$$

a) Find the electrostatic field?

(1-point)

$$\begin{aligned} \vec{E} &= -\nabla V = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \\ &= +4x\hat{a}_x + 3\hat{a}_y - 9\hat{a}_z \quad (\text{mV/m}) \end{aligned}$$

b) Determine the electrostatic force acting on a 1mC point charge located at point A(0,3,4)? (1-point)

$$\begin{aligned} \vec{F} &= q\vec{E} = \\ &= 1 \times 10^{-3} (4(0)\hat{a}_x + 3\hat{a}_y - 9\hat{a}_z) \\ &= 3\hat{a}_y - 9\hat{a}_z \quad (\mu\text{N}) \end{aligned}$$

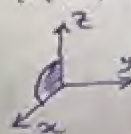
c) What is the volume charge density? Determine its value at a point B(1,0,0)? (0.5+0.5 point)

$$\begin{aligned} \rho_v &= \nabla \cdot \vec{D} = \epsilon_0 \nabla \cdot \vec{E} \\ &= \epsilon_0 \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] \\ &= \epsilon_0 (4 + 0 + 0) = 4\epsilon_0 \end{aligned}$$

$$\rho_v|_B = 4\epsilon_0 \quad (\text{mC/m}^2)$$

d) What is the electric flux passing through a circular surface of radius $1/\sqrt{\pi}$ located in yz-plane and centered at the origin? (1-point)

$$\begin{aligned} \psi &= \int \vec{D} \cdot d\vec{s} \\ &= D_y S \\ &= \epsilon_0 E_y S \\ &= 3\epsilon_0 * \pi \left(\frac{1}{\sqrt{\pi}}\right)^2 \\ &= 3\epsilon_0 \quad (\text{mC}) \end{aligned}$$

e) Calculate the electric flux passing through a cube of side 2m centered at the origin using two methods?Method-1

(0.5 point)

$$\begin{aligned} \psi &= Q_{\text{enc}} = \int \rho_v dv \rightarrow \nabla \cdot \vec{D} \\ &= \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 4\epsilon_0 dx dy dz \\ &= 4\epsilon_0 (2)(2)(2) \\ &= 32\epsilon_0 \quad (\text{mC}) \end{aligned}$$

Method-2

(0.5 point)

$$\begin{aligned} \psi &= \oint \vec{D} \cdot d\vec{s} \\ &= \int_{-1}^1 \int_{-1}^1 D_x|_{x=+1} dy dz - \int_{-1}^1 \int_{-1}^1 D_x|_{x=-1} dy dz \\ &\quad + \int_{-1}^1 \int_{-1}^1 D_y|_{y=+1} dx dz - \int_{-1}^1 \int_{-1}^1 D_y|_{y=-1} dx dz \\ &\quad + \int_{-1}^1 \int_{-1}^1 D_z|_{z=+1} dx dy - \int_{-1}^1 \int_{-1}^1 D_z|_{z=-1} dx dy \\ &= \epsilon_0 [4(2)(2) + 4(2)(2)] \\ &= 32\epsilon_0 \quad (\text{mC}) \end{aligned}$$

f) Determine the potential difference between A and B using two methods? (0.5 point)

A(0,3,4)
B(1,0,0)

(0.5 point)

Method-1

$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= [0 - 9 + 36] - [-2 + 0 + 0] \\ &= 29 \text{ V} \end{aligned}$$

Method-2

$$\begin{aligned} V_{AB} &= - \int_0^A \vec{E} \cdot d\vec{L} \\ &= - \left[\int_0^1 E_x dx + \int_0^3 E_y dy + \int_0^4 E_z dz \right] \\ &= - [-2 + 9 - 36] \\ &= 29 \text{ V} \end{aligned}$$

g) Calculate the work done needed to move a unit positive charge from point A to point B? (1-point)

$$\begin{aligned} W_{AB} &= Q V_{AB} \\ &= 29 \text{ J} \end{aligned}$$

h) What is the energy density at A? (1-point)

$$\begin{aligned} w_E &= \frac{1}{2} \epsilon_0 |\vec{E}_A|^2 \\ &= \frac{1}{2} \epsilon_0 [3^2 + 9^2] \\ &= \frac{1}{2} \epsilon_0 [81 + 81] \\ &= 45 \epsilon_0 \text{ J/m}^3 \end{aligned}$$

i) What is the work done needed to move a point charge in a rectangular loop taking path (1,0,0) → (1,2,0) → (1,2,1) → (1,0,1) → (1,0,0)? Discuss your result? (1-point)

$$\oint \vec{E} \cdot d\vec{L} = 0$$

- Work done in moving a charge around a closed path is equal to zero.

- This is the principle of conservative field.

j) Find the electrostatic energy stored in a cube of side 2m centered at the origin? (1-point)

$$\begin{aligned} W_E &= \frac{1}{2} \epsilon_0 \int |\vec{E}|^2 dV \\ &= \frac{1}{2} \epsilon_0 \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 [16x^2 + 9 + 81] dx dy dz \\ &= \epsilon_0 \left[80(2)(2)(2) + 8 \frac{2}{3} (2)(2) \right] \\ &= \left(540 + \frac{64}{3} \right) \epsilon_0 \text{ J} \end{aligned}$$



Tanta University
Faculty of Engineering
Electrical Power and Machines Engineering Dept.



Final Exam – First Semester 2015-2016

Course: EPM2104/EPM2141(Electromagnetic Fields)

Time allowed: 3 hr

Year: 2nd Electrical Power/Communications

Date: 27/1/2016

No. of Pages: 4

Total Score: 85


Remarks: Attempt to solve all of the following questions

Question 1

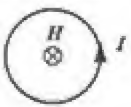
10 Points

Choose the correct answer for the following statements: (Verification of your choice is A MUST when numerical data are given)


- (1) Plane $z = 10$ m carries charge 20 nC/m^2 . The electric field intensity at the origin is
 (a) $-10a_z \text{ V/m}$ (b) $-18\pi a_z \text{ V/m}$ (c) $-72\pi a_z \text{ V/m}$ (d) $-360\pi a_z \text{ V/m}$
- (2) Point charges 30 nC , -20 nC , and 10 nC are located at $(-1,0,2)$, $(0,0,0)$, and $(1,5,-1)$, respectively. The total flux leaving a cube of side 6 m centered at the origin is:
 (a) -20 nC (b) 20 nC (c) 10 nC (d) 30 nC
- (3) A potential field is given by $V = 3xy - 5y$. Which of the following is not true?
 (a) The potential difference between point $(2, -1, 4)$ and point $(2, -1, -4)$ is zero.
 (b) At point $(1, 0, -1)$, E vanish.
 (c) The electric field at $(2, -1, 4)$ is $3a_x - a_y \text{ V/m}$.
 (d) The potential at $(0, 1, 0)$ is -5 V .
- (4) Which is not an example of convection current?
 (a) Electric current flowing in a copper (b) A beam of moving charges
 (c) Electronic movement in a vacuum tube (d) An electron beam in cathode ray tube
- (5) The relaxation time of a material having $\sigma = 10^{-17} \text{ mho/m}$ and $\epsilon_r = 5$ is
 (a) 5×10^{-10} seconds (b) 10 minutes (c) 15 hours (d) 51.2 days
- (6) A capacitor connected to a battery stores energy twice as much with a given dielectric as it does with air. The susceptibility of the dielectric is
 (a) 0 (b) 2 (c) 1 (d) 3
- (7) Identify the configuration in the figure that is not a correct representation of I and H
 (a) Configuration (b) Configuration (c) Configuration (d) Configuration



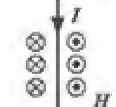
(1)



(2)



(3)



(4)
- (8) Two parallel wires carry currents along the same direction. The force experienced by one due to the other is
 (a) Perpendicular to the lines and (b) Parallel to the lines
 (c) Perpendicular to the lines and repulsive (d) Zero
- (9) The flux through each turn of a 100-turn coil is $(t^3 - 2t) \text{ mWb}$, where t is in seconds. The coil induced emf at $t = 2$ second is
 (a) 1 V (b) -1 V (c) 4 mV (d) -4 mV
- (10) Identify which of the following expressions is (are) not Maxwell's equations for time-varying fields:
 (a) $\nabla \cdot J + \partial \rho_v / \partial t = 0$ (b) $\nabla \cdot E = -\partial B / \partial t$
 (c) $\nabla \cdot D = \rho_v$ (d) $\oint H \cdot dl = \int (\sigma E + \epsilon \partial E / \partial t) \cdot ds$

Question 2**10 Points**

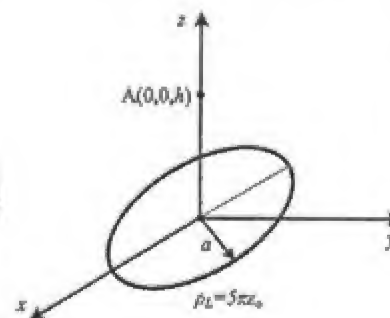
State true (✓) or false (×) and correct the false statements

- (1) Both ϵ_0 and χ_e are dimensionless.
- (2) The electric flux density on a spherical surface $r = b$ produced by a point charge Q located at the origin is the same as that produced by a charge of the same value as Q but distributed over the surface $r=a$ where $a < b$.
- (3) Inside a conductor, the electric field intensity is changes with the position.
- (4) A conductor is an equipotential body.
- (5) For a free-charged dielectric-dielectric interface, the tangential components of the electric flux density in the two materials are equal.
- (6) Faraday's law states that the line integral of the tangential component of H around a closed path equals the net current enclosed by the path.
- (7) An isolated magnetic pole exists.
- (8) The magnetic vector torque (T) on a current loop placed in a magnetic field is the vector product of its magnetic moment m and the magnetic flux density B .
- (9) For any solid cylindrical conductor, the magnetic field inside the conductor does not contribute to its total inductance.
- (10) Sometimes a voltage is induced across a conductor when it remains absolutely stationary within a steady magnetic field.

Question 3**15 Points (1,3,3,[2,2,2,2])**

A circular ring of charge with radius a lies in free space in the $z = 0$ plane, centered at the origin and has a uniform charge density of ρ_L C/m.

- (a) What is the total charge of the ring
- (b) Find the electric field intensity at point $A(0,0,h)$
- (c) Find the electric potential at point A
- (d) For $\rho_L = 5\pi\epsilon_0$ C/m and $a = 4$ m. Calculate:



- (i) The magnitude of electric field intensity at point $P_1(0,0,3)$
- (ii) The force acting on a unit positive-charge placed at $P_2(0,0,-3)$
- (iii) The work-done needed to move a unit positive charge from point P_1 to point P_2
- (iv) If a point charge Q_1 is located at the origin, Find the value of Q_1 which produce the same field intensity of the ring at point P_1

Question 4**15 Points (3,[4,2],[3,3])**

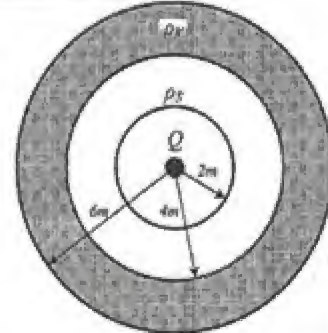
(a) Starting with Gauss's law, deduce an expression for electric field intensity of a point charge

(b) Consider the following charge distribution:

- a point charge of $10\mu\text{C}$ is located at $r = 0$, and
- a uniform surface charge density of $-1\mu\text{C}/\text{m}^2$ at $r = 2$

(i) Calculate the electric flux density D at $r = 1$ and $r = 3$

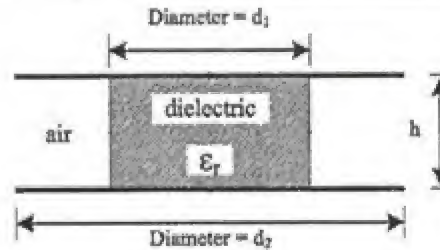
(ii) What uniform volume charge density should be established in the region $4 < r < 6$ for D to vanish at $r = 7$.



(c) For the capacitor shown in the Figure.

Find expressions for the following

- (i) the capacitance
- (ii) the energy density in each region

**Question 5****16 Points (2,4,4,2,2,2)**

An infinitely long straight filament carries current of (I) lies in free-space along z -axis.

- (a) Use Ampere's law to obtain the magnetic field intensity and the magnetic flux density at point $(0,4 \text{ meters}, 0)$.
- (b) Determine the force exerted on the filament if the area surrounding it has a magnetic flux density of $\vec{B} = \hat{a}_x - \hat{a}_y \text{ T}$.
- (c) Determine the mutual inductance between the filament and a single turn rectangular coil of sides a and b placed on the x - z plane with the one of the b sides lies along the x -axis between the points $(x_0, 0, 0)$ and $(x_0 + b, 0, 0)$.
- (d) If the filament current varies sinusoidally with time as $I_m \sin(\omega t)$.
 - (i) Find the emf induced in the coil as well as the integral of the produced motional electric field dot $d\vec{l}$ along the turn perimeter.
 - (ii) Write the appropriate Maxwell's equation for the previous case in the integral form. Hence derive its differential form in not more than three steps of answer.
 - (iii) If the relative permeability of the medium surrounding the filament is 50 and the filament carries a dc current I , determine the magnetic polarization (magnetization).

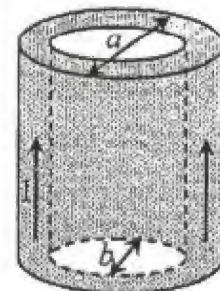
Question 6**19 Points ([7,2,2],[2,6])**

(a) Consider an infinite length hollow conducting tube of conductivity σ_1 S/m carrying a current I with a uniform current density as shown in the figure.

- (i) Apply Ampere's law to derive expressions for the magnetic field intensity everywhere and sketch the results as a function of the radius r

- (ii) Derive a formula for the resistance per unit length of the tube

- (iii) The space $0 < r < b$ is now filled with a conducting material whose conductivity is σ_2 S/m. Current I in Ampere, flows through the area $0 < r < a$ with a constant current density. Derive a formula for the voltage drop across each unit length of the filled tube



(b) A point charge of $4\pi\epsilon_0$ mC is located at point (10,-1,1) in Cartesian coordinates in the presence of a perfectly conducting plane located at $z=0$ in free space.

- (i) Sketch the image equivalence and the electric field lines
- (ii) Calculate the electric field intensity, the electric potential and the surface charge density at (0,0,0).

Wish you all the best



You may substitute for the following constants, Only, if is needed to add terms containing them. Otherwise, leave the results in terms of π , ϵ_0 and/or μ_0 .

$$\pi = 3.14159$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$$

Question 1:

Marks [25]

a- State true or false and correct the false statements

- 1- The displacement current is significant at low frequencies.
- 2- The choice of Gauss's surface depends on the symmetry of the charge distribution in space.
- 3- The electric field at the boundaries between two dielectric materials has normal components only. *E tangent components*
- 4- Gas conductivity depends only on the motion of negative charges. *(+ve & -ve)*
- 5- The charge inside a conductor is not zero in steady state.

b- A charge of 2 nC/m^2 is uniformly distributed over the infinite plane that passes through the points $(2, 2, 0)$, $(2, -2, 0)$ and $(0, 0, 1)$ in Cartesian coordinates. Find the vector of the electric field produced.

c- Aided with equation(s), define the following terms:

- | | | |
|--|---|---|
| $\vec{P} = \chi \vec{E}$ $\vec{E} = \frac{\rho}{\epsilon_0}$ $\vec{E} = -\nabla V$ | σ Conductivity - $\sigma \nabla V$ | \vec{J}_D Displacement current density |
| Polarization D-D | Voltage gradient | Continuity of current |
| Divergence theorem | | |

Question 2:

Marks [20]

a- Three infinite uniform sheets of charge are located in free space as follows:

$$3 \text{ nC/m}^2 \text{ at } x = -4, 6 \text{ nC/m}^2 \text{ at } x = 1, \text{ and } -8 \text{ nC/m}^2 \text{ at } x = 4.$$

In addition, a point charge of 2 nC at $P_Q(x=2, y=0, z=0)$ and a line charge of -2 nC at $y=3, z=3$. Find \vec{E} at the point:

(a) $P_A(2, 2, 2)$; (b) $P_B(0, 0, 0)$, in Cartesian coordinates

b- Given the electric flux density $\vec{D} = 0.3r^3 \vec{a}_r$, nC/m^2 in free space:

- (a) find electric field strength at point $P(r=2, \theta=25^\circ, \phi=90^\circ)$;
- (b) find the total charge within the sphere $r=4$.

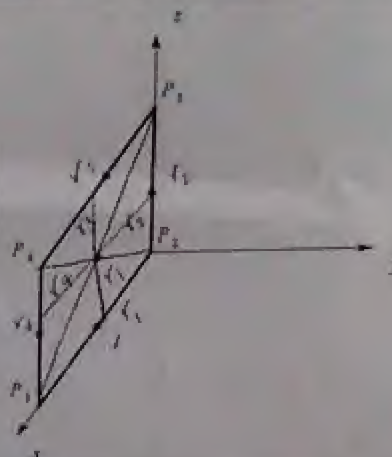
c- A parallel-plate capacitor contains three dielectric layers of 20 cm^2 are. The layers has the given relative permittivity and width of $\epsilon_1=1$ and $d_1=0.2 \text{ mm}$, $\epsilon_2=2$ and $d_2=0.3 \text{ mm}$ and $\epsilon_3=3$ and $d_3=0.4 \text{ mm}$. Find the overall capacitance and the percentage of the total stored energy located in each of the three regions. If a voltage of 100 Volt is applied to the capacitor terminals. Find the total polarization in first layer.



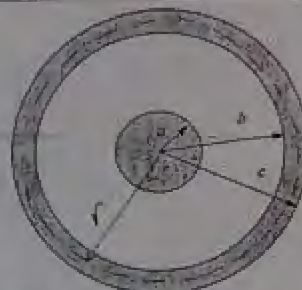
Question 3:

Marks[20]

- a- Find the magnetic field intensity \vec{H} in Cartesian coordinate system at P_5 , the center of the rectangle $P_1(10,0,0)$, $P_2(0,0,0)$, $P_3(0,0,8)$ and $P_4(10,0,8)$. The field is produced by filamentary current of 10A circulating as shown in figure.



- b- Express the value of \vec{H} in Cartesian components at $P(r=0.01, \phi=0, z=0)$ in the field of coaxial cable $a=3\text{mm}$, $b=9\text{mm}$, $c=12\text{mm}$ and $I=0.8\text{A}$. The current follows axially into the central conductor, the positive z direction.



- c- Find the current density vector causing the magnetic field strength:

$$\vec{H} = xyz(\vec{a}_x + \vec{a}_z)$$

Question 4:

Marks[20]

- a- Write down Maxwell equations for magneto static field and static electrical field in free space in both differential and integral forms.
- b- Two single turn coils A and B are so positioned that 60% of the flux produced by B links coil A. A current of 6A in coil B produces a total flux of 1.8mWb. Determine:
[1] the mutual inductance.
[2] the e.m.f. induced in coil A when a current of 6A in coil B is reduced linearly to zero in 1ms.
- c- Derive an expression for the voltage generated by a Faraday disc of radius b and rotates with an angular velocity ω radians/sec in a uniform magnetic field of density B Tesla.